SIMULATION OF DYNAMIC QUALITIES OF SIMPLE AIRCRAFT PNEUMATIC SYSTEMS

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Summary: *The paper deals with modelling of simple pneumatic systems by means of the theory of electro-pneumatic analogy. The aircraft pitot-static system was described as an electric circuit and its transfer function was calculated using the common electric circuit analytic method.*

I. FORMULATION OF THE PROBLEM

Though the aircraft pitot-static system belongs to the most simple board systems, it is necessary to pay great attention to it during the aircraft operation as its wrong function can essentially influence the flight security. Almost in all prescribed works and preparations, inspections of the state of individual parts of the system and tests of the clearness and tightness are carried out. Also the dynamic behaviour of the pitot-static system is an important matter. It is predetermined both by its construction, and the temperature and density of the air which the pitot-static system is filled with.

The pitot-static system regularly consists of two air pressure sensors (the main and the reserve one), the piping circuit, the instruments and gears to which these pressures are supplied and some additional components, e.g. water separators, the switch of the main and reserve pitot-static tube, the switch of the hypersonic and subsonic system of the static pressure etc. A typical pitot-static system of the military aircraft of the sixties is presented in the Figure No. 1.

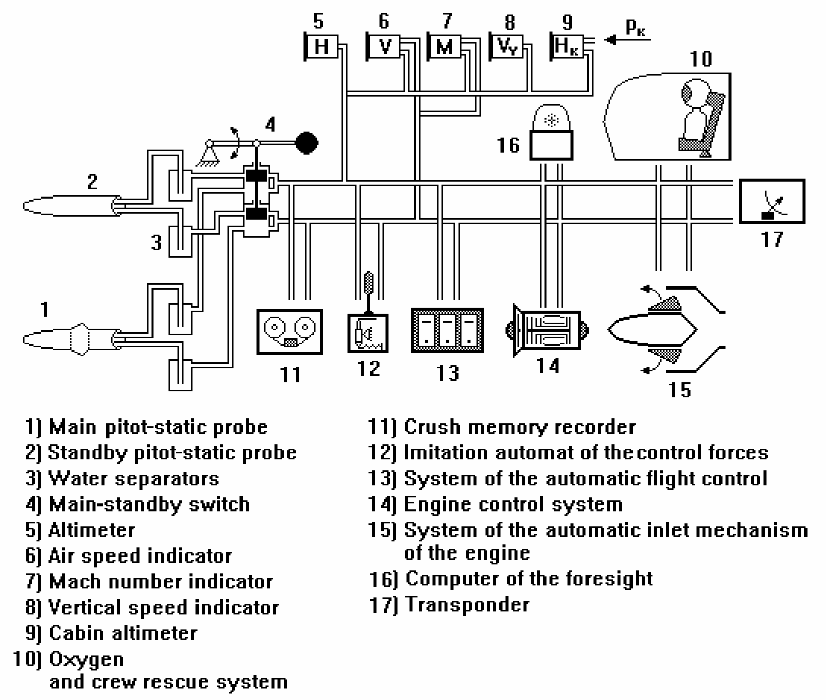

Fig. 1: Aircraft pitot-static system of the sixties

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The pitot-static systems of the military aircraft of the sixties were very spacious and therefore also bulky, heavy and relatively unreliable (problems with tightness and clearness). Contemporary military aircraft are equipped with much less spacious pitot-static systems thanks to the fact that the air data computers have been established which replace a substantive part of the piping circuit with the electric one.

The dynamic qualities of the pitot-static system or – more explicitly – the delay caused by it can, under certain conditions, reach some very significant values. If the changes of the dynamic qualities of the pitot-static system are not respected, e.g. in extensive renovating reconstructions, some problems both in manual and in automatic flight control can appear.

In a simplified view, the manual flight control, possibly the control of one flight parameter can be described as an action of a separate control loop consisting of three dynamic members: pilot, aircraft and instrument.

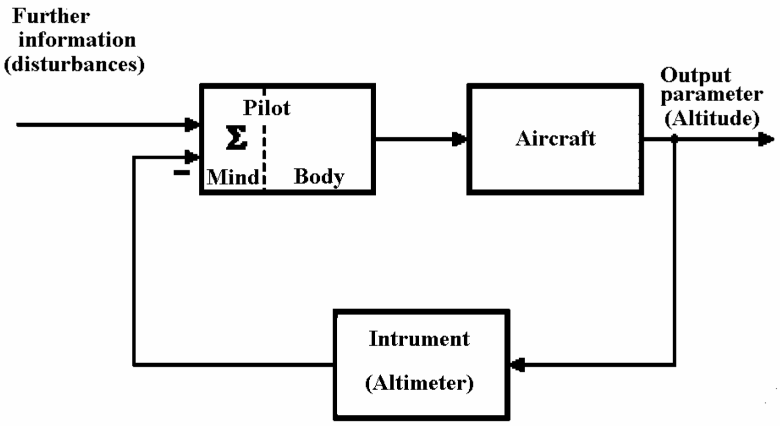

Fig. 2: Loop „Pilot – Aircraft – Instrument“

From the point of view of its dynamic qualities, it is necessary to view the proper instrument – barometric altimeter – as two connected dynamic members, i.e. the pitot-static system and the altimeter itself which represents the feedback of the control loop. Therefore both the altimeter and the pitot-static system essentially influence the quality of the flight control.

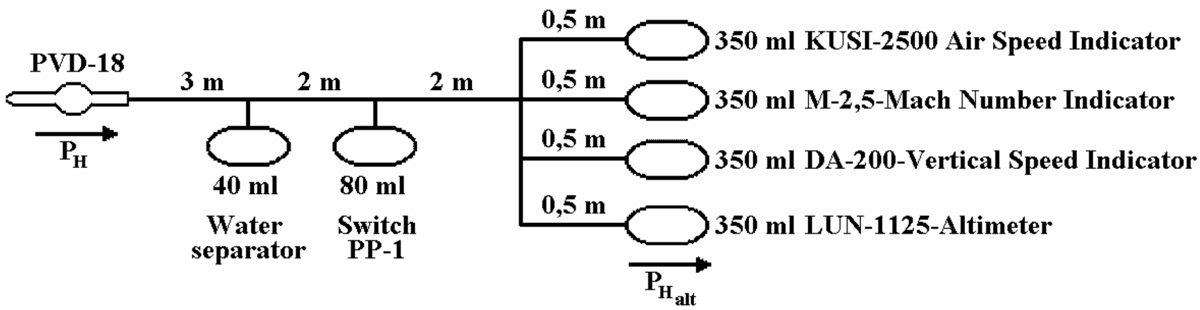

Fig. 3: Constructional parameters of the subsonic static system

If we want to expect the dynamic qualities of the pitot-static system before its implementation or a substantial structure transformation, it is reasonable to use a computer simulation e.g. on the basis of electropneumatic analogy.

As an example, it is possible to have the pitot-static system presented in the Figure No. 3.

To create an accurate mathematical model of a dynamic object, it is necessary to define its physical, possibly constructional parameters as accurately as possible. In this case, especially the lengths and inner diameters of individual sections of the piping circuit and the extents of individual instruments connected to the piping circuit are considered.

It is obvious that in our model, there are omitted the construction details of the pitot-static system as pneumatic resistance of the static-pressure openings of the pitot-static tube, reducers of the hose-pipe tube, the switch “main/reserve pitot-static tube”, the differences in pneumatic resistance of the tube mouths into the vessels during the in- and outflow of the gas etc.

Taking into consideration that some of the pneumatic resistances have been neglected, the real processes proceeding in the pitot-static system are probably slower than the modelled ones.

II. THEORY OF THE ELECTROPNEUMATIC ANALOGY

This theory enables to transfer the calculation of dynamic qualities of pneumatic or hydraulic arrangements to the treatment of electric arrangements on the basis of physical similarity – analogy. On this basis work – or better said: worked – analogue computers.

To create an electric scheme in its dynamic behaviour corresponding with the dynamic behaviour of the pitot-static system, the theory of the electropneumatic analogy can be used.

According to this theory, the physical magnitude of air pressure can be compared to the electric magnitude of voltage and the capacity convection of the air to the electric current.

2.1 Pneumatic resistance:

The relation between the pressure decrease on the tube in which the gas flows can be compared with the voltage decrease on the resistor through which the electric current flows.

To get the pressure decrease Δp on the tube of round crosscut with the diameter d and length l flown by the capacity convection of the gas Q the dynamic viscosity of which is μ, it is possible – in case of laminar convection (Reynolds’ number $\mathrm{Re} \leq 2300$) – to write the formula:

$$\Delta p = \frac{128}{\pi} \cdot \mu \cdot \frac{l}{d^4} \cdot Q \ , \qquad (1)$$

possibly the formula for the pneumatic resistance:

$$R_{pn} = \frac{\Delta p}{Q} = \frac{128}{\pi} \cdot \mu \cdot \frac{l}{d^4}, [\mathrm{Pa} \cdot \mathrm{m}^{-3} \cdot s] \; . \tag{2}$$

The pneumatic resistance depends also on dynamic viscosity of the air which represents a function of the temperature. Its temperature dependence can be described by the formula:

$$\mu_t = \mu_0 \cdot \sqrt{1 + 3{,}665 \cdot 10^{-3} \cdot t} \cdot \left(1 + 8 \cdot 10^{-4} \cdot t\right)^2 \; [\mathrm{Pa} \cdot \mathrm{s}], \; \mu_0 = 1{,}712 \cdot 10^{-6} \, \mathrm{Pa} \cdot \mathrm{s} \quad ,$$

or

$$\mu_t \; = 1{,}71201 \cdot 10^{-6} \cdot \left(1{,}88205 \cdot 10^{-6} \cdot t^2 + 3{,}43619 \cdot 10^{-3} \cdot t + 1\right)$$

where t je Celsius' temperature ($\mu_{(t=15^o C)} = 1{,}8 \cdot 10^{-6} \, Pa \cdot s$).

The matter is more complicated with the turbulent convection of the air ($\mathrm{Re} > 2300$). The pressure decrease on the piping depends of the rate of flow nonlinearly, moreover it depends also of the pressure ratio before and behind the tube, i.e. of the fact whether the subcritical or above critical convection arises.

In our case, the condition of convection laminarity will be accomplished; to calculate the pneumatic resistances, the formula No. 2 will be used. To describe the pneumatic resistance of the tube during turbulent convection, an approved formula can be found in professional literature:

$$\Delta p = p_2 - p_1 = 1{,}6 \cdot 10^3 \cdot \frac{l}{d^5} \cdot \frac{1}{p_1} \cdot Q^{1{,}85} \quad . \tag{3}$$

2.2 Pneumatic induction:

The difference of pressure before and behind the piping caused by the influence of inertia of the gas mass (its momentum) at time changes of the capacity convection can be compared to the voltage on the induction at time changes of electric current which flows in it.

If the gas of the density ρ flows through the piping with the diameter d and length l, the following formula for the pressure difference before and behind the piping caused by the time change of the capacity convection can be written:

$$\Delta p(t) = \frac{4}{\pi} \cdot \rho \cdot \frac{l}{d^2} \cdot \frac{dQ(t)}{dt}, \tag{4}$$

or also the formula:

$$Q(t) = \frac{\pi}{4} \cdot \frac{1}{\rho} \cdot \frac{d^2}{l} \int_0^t \Delta p(t) \cdot dt \, . \tag{5}$$

The pneumatic induction can be therefore described in this way:

$$L_{pneum} = \frac{4}{\pi} \cdot \rho \cdot \frac{l}{d^2}, [\mathrm{Pa} \cdot m^{-3} \cdot s^2] \, . \tag{6}$$

2.3 Pneumatic capacity:

The behaviour of the vessel into which the capacity convection of the gas is brought, can be, similarly, compared with the behaviour of the electric capacity into which the electric current is brought.

The gas compression in the instrument vessels as well as in the piping circuit is assumed to be an adiabatic process (Poisson's constant $k = 1{,}4$). In accordance with this presumption, also the calibrating equations of aircraft instruments are derived.

To get the capacity conduction of the gas Q with the density ρ and the absolute temperature T which flows into the vessel with the capacity V as a consequence of the time change of the pressure, following formula:

$$\mathrm{Q}(t) = \frac{V}{\rho \cdot k \cdot R \cdot T} \cdot \frac{d\Delta p(t)}{dt} \,, \tag{7}$$

or the formula:

$$\Delta p(t) = \frac{\rho \cdot k \cdot R \cdot T}{V} \cdot \int_0^t Q(t)dt \,, \tag{8}$$

can be written where:

k - Poisson's constant,
ρ - air density,
R - gas constant of the air,
V - capacity of the respective instrument or circuit section,
T - absolute temperature of the air.

Therefore, the pneumatic capacity can be defined by following formula:

$$C_{pneum} = \frac{V}{\rho \cdot k \cdot R \cdot T} \,, [Pa^{-1} \cdot m^3] \,. \tag{9}$$

III. DESIGN OF A MODEL ELECTRIC SCHEME OF THE PITOT-STATIC SYSTEM

By means of the formulas to calculate electric analogies of the pneumatic resistances, inductions and capacities, a quite new model of the pitot-static system has been created, see Figure No. 4a, and also a simplified model in which the capacities of the instruments and their supplying hoses are connected into one simple RC member, see Figure No. 4b.

For dimension reasons, the complete admittance matrices of the models a) and b) are not presented.

In the simplest model (see Figure No. 4c), the inductions of the piping have been neglected. The complete admittance matrix of the model c) stays in the form:

$$
Y_C = \begin{bmatrix}
G_1 & -G_1 & 0 & 0 & 0 & 0 \\
-G_1 & \left(G_1 + pC_{so1} + G_2\right) & -G_2 & 0 & 0 & -pC_{so1} \\
0 & -G_2 & \left(G_2 + pC_{so2} + G_3\right) & -G_3 & 0 & -pC_{so2} \\
0 & 0 & -G_3 & \left(G_3 + pC_{o3} + G_4\right) & -G_4 & -pC_{o3} \\
0 & 0 & 0 & -G_4 & \left(G_4 + pC_{s3}\right) & -pC_{s3} \\
0 & -pC_{so1} & -pC_{so2} & -pC_{o3} & -pC_{s3} & \left(pC_{so1} + pC_{so2} + pC_{o3} + pC_{s3}\right)
\end{bmatrix}.
$$

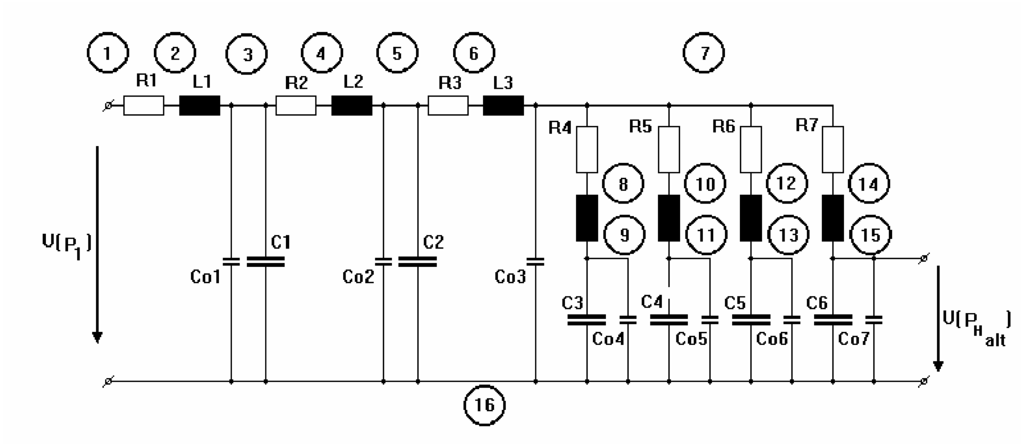

Fig. 4a: The schemes of electric analogies of the pitot-static system

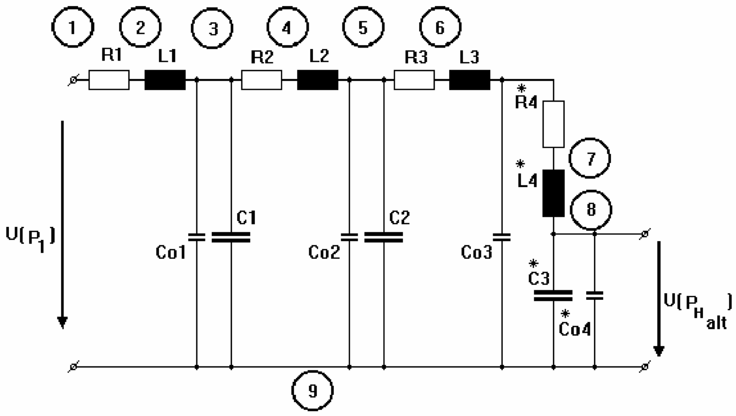

Fig. 4b: The schemes of electric analogies of the pitot-static system

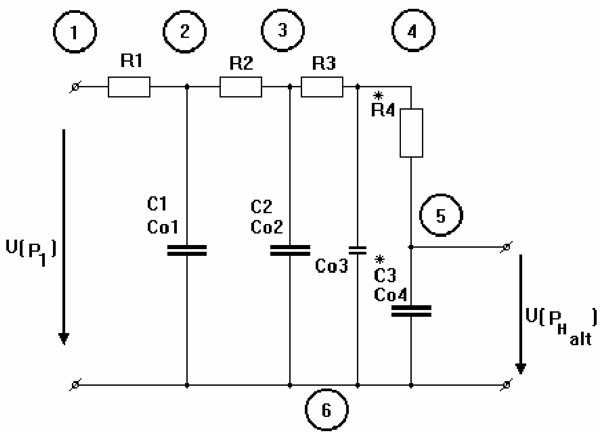

Fig. 4c: The schemes of electric analogies of the pitot-static system

IV. CALCULATION OF THE TRANSITIONARY CHARACTERISTICS

The transfer functions of the individual model circuits were calculated by the common method of point-of-junction voltage in the Maple programme environment. For the component impedances, their Laplace's images have been input into the model.

By the scheme, the complete admittance matrix $\tilde{Y}$ has been designed by means of which the image of the transfer function has been calculated:

$$F(p) = (-1)^{\alpha+\beta} \cdot \frac{\Delta_{LN,LN}}{\Delta_{LN,MN}}, \tag{10}$$

where

$\Delta_{LN,LN}$ is the determinant of the matrix $\tilde{Y}$ in which the lines and columns corresponding to the input were omitted,

$\Delta_{LN,MN}$ - $\Delta_{LN,LN}$ is the determinant of the matrix $\tilde{Y}$ in which the lines and columns corresponding to the input and output were omitted,

α is the number of odd figures *L*, *M*, and *N*,

β is the number of descending pairs *LN MN*.

The Laplace's images of the transfer functions $F(p)$ were then multiplied by the image of the unit skip $F_{JS}(p) = 1/p$ and thus the images of the transfer characteristics were received. These images were then transformed back to the time area and graphically depicted.

V. THE RESULTS OF THE SIMULATION

The main experiments were carried out with the most complex model, e.g. with the variant a). By means of the unit skip, the influence of the altimeter capacity V and the inner diameter of the piping d upon the dynamic behaviour of the pressure inside the altimeter have been examined.

The transfer characteristics for the basic capacity of the altimeter 350 ml and the relative deviation from this capacity ±20 % a ±40 % were calculated.

The detected transfer characteristics correspond to the transfer characteristics of the overdamped oscillatory member and can be therefore also compared to the transfer characteristics of the inertial member of the 1st order with the time constant approx. 30 ms.

It can be seen from the graphic courses in the Figure No. 5 that the change of the altimeter capacity in the range ±40 % influences only inessentially the time constant of the air pressure in the altimeter. The time constant changes in the range approx. ±50 ms which represents only approx. ±13 %.

Another experiment examined the influence of the size of the inner diameter d of the pitot-static-system piping on the transfer characteristics. These characteristics were calculated for the diameters d =2,5 mm, 3,0 mm, 3,5 mm, 4,0 mm, 4,5 mm and 5,0 mm (see Figure No. 6). From the obtained characteristics, a great influence of the factor d is obvious.

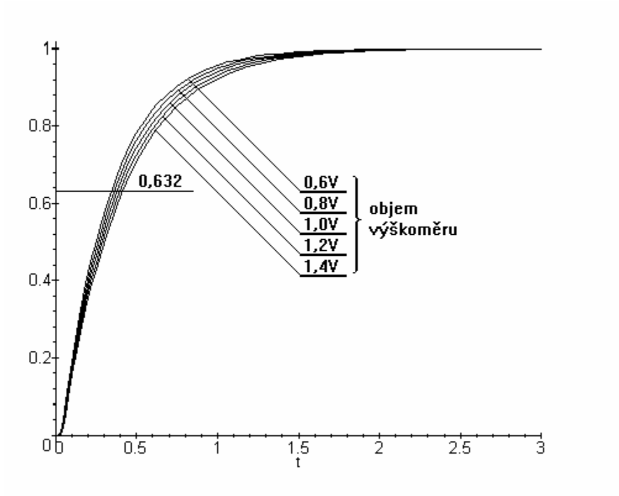

Fig. 5: Influence of the altimeter capacity upon the size of time constant

With the diameters 2,5 mm - 4,5 mm, the transfer characteristics correspond to the characteristics of the overdamped oscillatory processes and they can be compared to the transfer characteristics of the 1st-order inertial member with the time constants in the range of 100-1200 ms.

From the diagrams depicted in Figure No. 6, it is also obvious that the time constant rises exponentially in dependence on decrease of the inner diameter d of the piping.

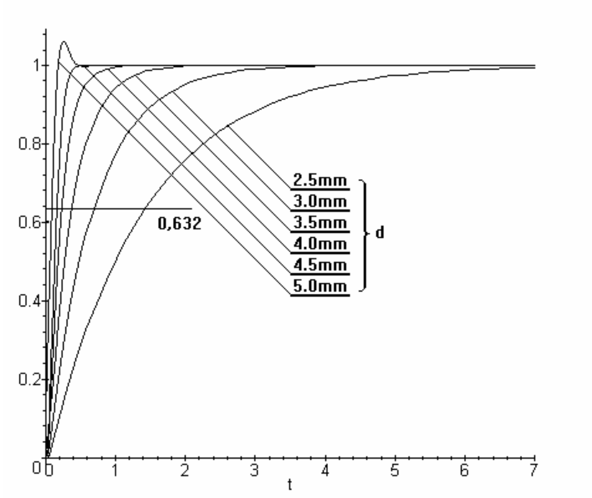

Fig. 6: Influence of the piping diameter on the transfer characteristics

With decrease of the diameter d from 3,5 mm to 2,5 mm, i.e. of approx. 30 %, the time constant will rise from 360 ms to 1200 ms, i.e. of more then 230 %.

With the inner diameter of the piping $d = 5$ mm, the transfer characteristics corresponds to the transfer characteristics of the oscillatory member.

Similarly to the preceding two cases, also the influence of the flight altitude on the transfer characteristics has been analyzed.

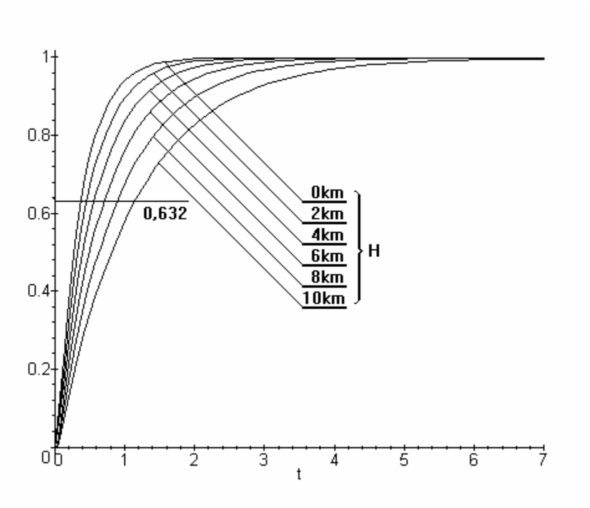

Fig. 7: Influence of the altitude on the time-constant increase

The calculation was carried out for standard atmospheric conditions characterized by pressure, density, temperature and dynamic viscosity of the air, for the altitudes 0 m, 2000 m, 4000 m, 6000 m, 8000 m and 10000 m.

From the transfer characteristics shown in the Figure No. 7, it can be seen that these characteristics correspond with the characteristics of the overdamped oscillatory member and they can be – in a simplified way – compared to the transfer characteristics of the 1st-order inertial member.

The time constant rises exponentially with the altitude. In 10 000 m, the time constant is approx. 1 100 ms, which represents – in comparison with the time constant 360 ms in 0 m – an increase of over 200 %.

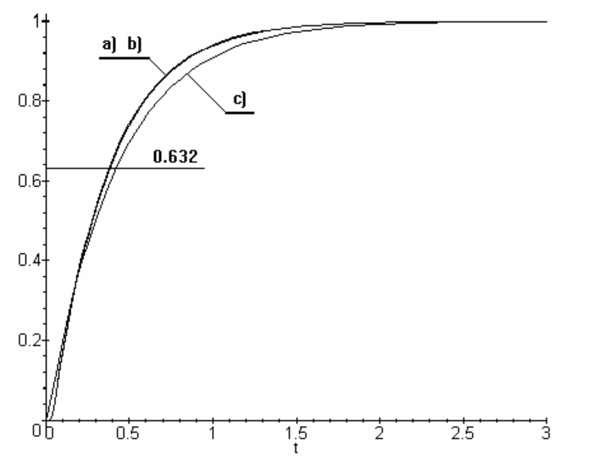

Figure No. 8: Comparison of the models

In the last experiment, the calculations made by means of the a), b), c) models were compared. The graphic courses of the transfer characteristics calculated under identical conditions (altitude – 0 m, altimeter capacity – 350 ml, inner diameter of the piping – 3,5 mm) are presented in the Figure No. 8. It can be seen from the diagrams that the transfer characteristics obtained by the a) and b) models are almost identical. Both of these models contain the influence of the pneumatic induction of the piping circuit and under given conditions they behave as massively overdamped oscillatory members the transfer characteristics of which are close to the transfer characteristics of the 1st-order inertial member.

The model c) represents the simplest model in which the influence of pneumatic induction of the piping circuit was neglected. The transfer characteristics of this simplest model corresponds to the transfer characteristics of the 1st-order inertial member.

It is obvious that even this much simplified model offers a quite good information of the reaction speed of the pitot-static system on the skip change of the input pressure, nevertheless only in the range of overdamped oscillatory processes.

VI. CONCLUSION

The paper presents a relatively simple mathematical-physical apparatus by means of which a quick computer analysis of the behaviour of simple pneumatic or hydraulic systems can be carried out, in our case of the static pressure subsystem of the aircraft pitot-static system.

In arrangement of the models and their following evaluation, it is necessary to consider the influence of the accepted constructional simplification of the pitot-static system, possibly the influence of qualified estimations of some of its parameters.

From obtained results of the analyzed model, following facts appear:

1) The mean of the time constant of the pitot-static system is 360 ms.

2) The change of altimeter capacity of +/-40 % (by unchanged length of the piping and its inner diameter) is demonstrated by the change of the transfer-process time constant of +/- 50 ms which amounts +/-13 %. Such a change of the time constant is inessential for the operation of the aircraft barometric instruments.

To affirm these theoretically obtained results, it would be efficient to carry out the measurement of the time constant of the pitot-static system on the aircraft.

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