OFFLINE PARAMETER IDENTIFICATION - THE RANDOM INITIAL ESTIMATION ISSUE

Michal Dub

University of Defence, Kounicova 65, 612 00 Brno, Czech Republic
Tel.: +420-973-445036, fax: +420-973-445174, e-mail: michal.dub@unob.cz

Abstract:

Experimental offline identification algorithm is based on minimization of criteria function considering the unknown mathematical model variables. We often use software support of various computing programs to find the minimum of a scalar function of several variables. However, operator has to know initial estimation of the searched parameters to get proper results. If operator sets bad initial estimation, the identification process would find another local minimum or would diverge. This paper is focused on random approach to initial estimation of simple mathematical model using basic MATLAB software support.

Keyword: identification, initial estimation, searched parameters, MATLAB

Introduction

Computer simulations have become a useful part of mathematical modelling of many natural systems in different areas of science and engineering. Simulations can be used to estimate the performance of systems and to explore new insights into new technology. Simulations are also integral parts of education process where lecturers often appreciate comfortable advantages of simulation programs.

A mathematical model is an abstract model that uses mathematical language to describe the behavior of existing or designed system. The mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. These models are classified into parametric and non-parametric models. Parametric models have given structure. The structure is a certain order and a chosen type of differential or difference equation, sets of these equations, transfer or z-transfer functions etc.

Identification is defined as a creation of a mathematical model for a given process. An identification based on mathematical and physical analyses is called analytical (clear box) identification. An identification based on measuring system data is called experimental (black box) identification. There are two methods of experimental identification, offline and online identification. This paper is exclusively focused on the offline identification method where the measuring of the system data is recorded and a mathematical model is created after the whole measuring process has finished.

Experimental offline identification algorithm is based on minimization of criteria function considering the unknown model variables. The problem is solved by searching for the function of several variables extreme [3][4]. Block scheme of the identification process is shown in the Fig. 1.

Simple Mathematical Model

Well know transfer function of the separately excited DC motor was chosen to get random approach to offline identification more familiar. It means we know the structure of the model and we only need to find parameters of this structure. Transfer function of the separately excited DC motor [2][3][4] can be simplified when no load torque is considered:

$$\omega(p) = \frac{K}{T_a T_m p^2 + T_m p + 1} u_a(p), \tag{1}$$

where ω - motor rotational speed,
u_a - motor armature voltage,
K, T_a, T_m - motor parameters.

The quality of the identification process is given by the difference of the real measured system response and the mathematical model simulated response. The criteria function J is used for the experimental offline identification, where the quality is assessed by the summation of square differences between system output ω_S and its model output ω_M:

$$J = J(x) = \sum_{i=1}^{N} \left[\omega_{Mi} - \omega_{Si}\right]^2 . \tag{2}$$

The model variables are motor gain coefficient and motor time constants which are stored in the vector of parameters $x = \begin{bmatrix} K & T_a & T_m \end{bmatrix}$. It is possible to represent the search of the parameters as search of the J function minimum.

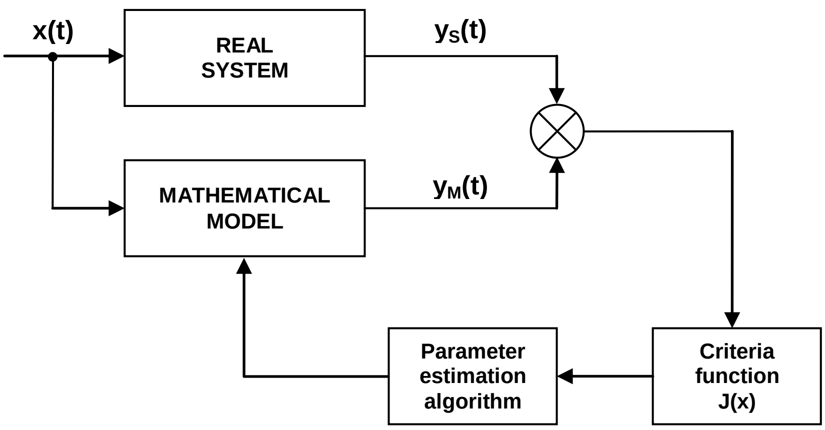


Fig. 1: Block scheme of the identification process

The software support of the *fminsearch* function of the MATLAB program was used to find the minimum of a scalar function of several variables. The identification algorithm is the Nelder-Mead simplex search method. It is a direct search method that does not require gradients or other derivative information. If n is the length of x, a simplex in n-dimensional space is characterized by the n+1 distinct vectors which are its vertices. In two-space, a simplex is a triangle; in three-space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated.

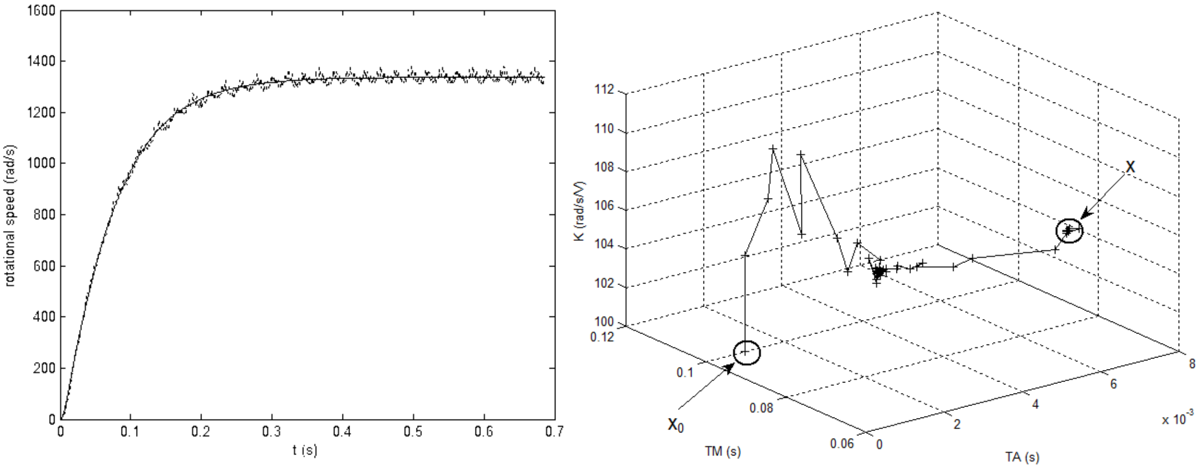
The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance [5].

Knowledge Based Parameterization of the Mathematical Model

System data are featured by input armature voltage and output rotational speed and armature current. The nominal voltage step function was used as a test input function. Rotational speed was measured by DC tachogenerator and armature current was measured by oscilloscope current probe. We used TiePie-HS4 oscilloscope with sampling frequency set to 20 kHz to measure and record the data. The Handyscope HS4 features a user selectable 12-16 bit resolution, 200 mV - 80 V full scale input range, 128 Ksamples record length per channel and a sampling frequency up to 50 MHz on all four channels. Connected to the fast USB 2.0 interface, the Handyscope HS4 doesn't require an external power supply.

Rotational speed was measured by K6A1 DC tachogenerator. Firstly we had to find conversion ratio rotational speed to tachogenerator voltage. Input rotational speed was measured by HHT11-R non-contact digital tachometer and the output tachogenerator voltage was measured by a digital voltmeter. Measured tachogenerator voltage was converted to the rotational speed using MATLAB computation functions. Noise fast changes of rotational speed were filtered off using low-pass filter with 0.003 sec time constant to eliminate sensor-processing errors.

Experienced operator would set initial estimate x_0 of the searched parameters according to the system measured response as shown in the Fig. 2. The input armature voltage step was about 12 Volt, so the motor gain (K) was set to 100 rpm/Volts. The electromechanical time constant (T_m) was estimated less than one tenth of a second and electromagnetic time constant (T_a) was set to one millisecond.



x_0=[100 0.001 0.1]

x =[1.0654e+002 6.0102e-003 6.8055e-002]

Fig. 2: Example of the experience identification (good results)

Rotational speed responses of measured and simulated system to voltage armature step are shown in Fig. 2 on the left. Rotational speed rip of the measured system is based on tachogenerator voltage rip. Searching of parameters is shown in Fig. 2 on the right.

Random Choice of the Initial Estimation

Inexperienced operator has to follow trial and error method. It is possible to use random generator to simulate inexperienced operator effort (only positive numbers are used). Results of the random simulations are shown in Fig. 3 and listed below in Table 1. Examples of bad and good identification processes including criteria function (on the left) and vector of parameters iteration progress (on the right) are explored in Fig. 4 and Fig. 5.

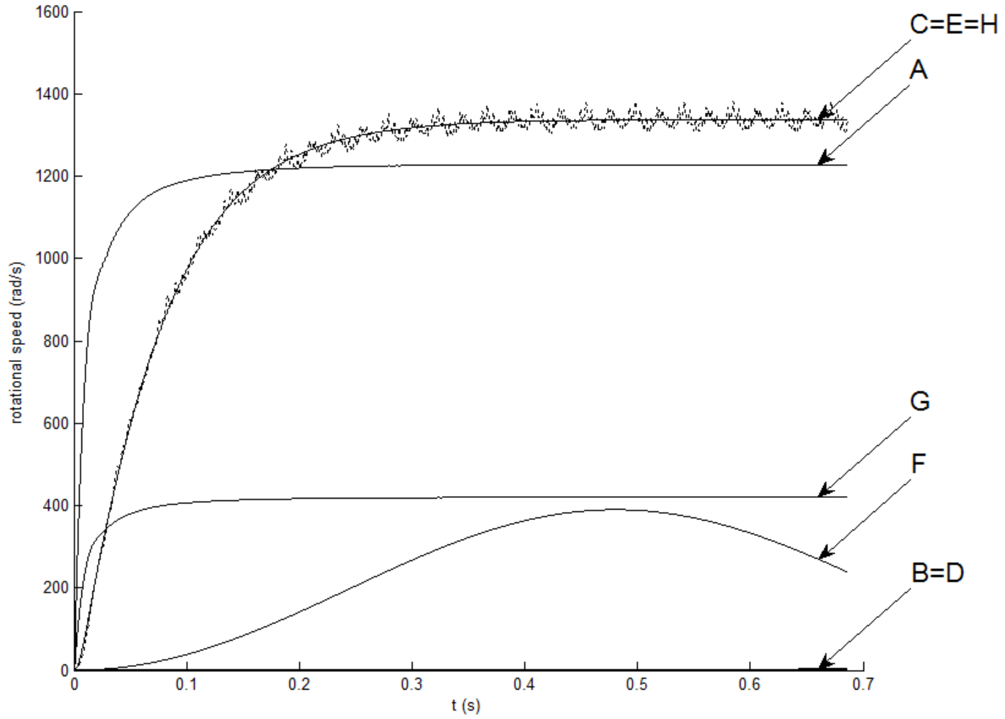
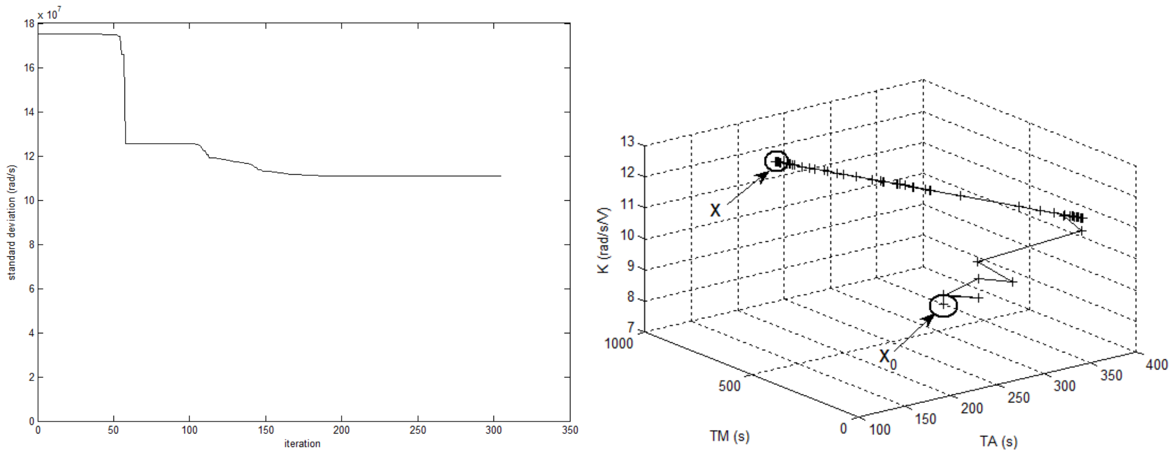
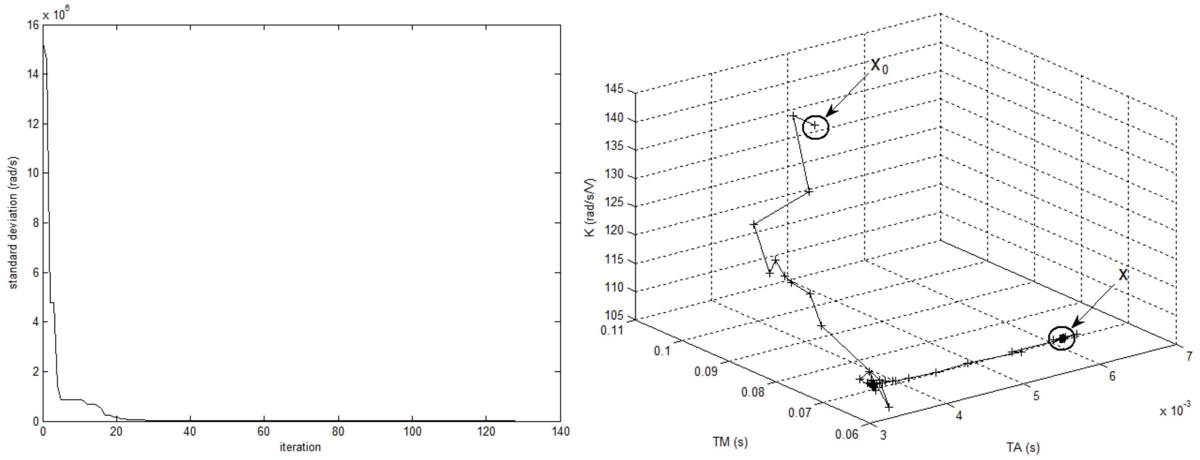


Fig. 3: Example of the identification with random initial estimates (using final vectors x from Table 1)



x_0=[6.4432e+000 3.7861e+002 8.1158e+002];
x =[1.5864e+001 1.0731e+001 2.1056e-003]

Fig. 4: Example of the random identification (bad results – case F)



x_0=[1.4189e+002 4.2176e-003 9.1574e-002];
x =[1.0654e+002 6.0102e-003 6.8055e-002]

Fig. 5: Example of the random identification (good results – case H)

Table 1: Identification process with random initial estimates

	x_0	x	J	σ
A	2.3478e+003 3.5316e+000 8.2119e-004	9.7827e+001 4.1029e-012 9.5298e-003	6.1831e+008	2.1214e+002
B	6.4912e-004 7.3172e-004 6.4775e-003	3.9213e+003 2.9453e-007 1.0479e+004	2.0470e+010	1.2206e+003
C	7.4469e-001 1.8896e+000 6.8678e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
D	7.8023e-003 8.1126e-003 9.2939e+001	2.9786e-002 1.4805e-014 5.7847e-011	2.0514e+010	1.2219e+003
E	4.4678e+002 3.0635e-001 5.0851e-001	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
F	6.4432e+000 3.7861e+002 8.1158e+002	1.5864e+001 1.0731e+001 2.1056e-003	1.3001e+010	9.7276e+002
G	8.7594e+000 5.5016e-002 6.2248e+003	3.3438e+001 2.0093e-010 9.8757e-003	9.2765e+009	8.2170e+002
H	1.4189e+002 4.2176e-003 9.1574e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001

It is useful to repeat simulations listed in Table 1 with the same order of magnitude of the initial estimate to explore each identification process in detail. Simulations with the worst initial results (cases B and D) are listed in Table 2 and Table 3. Simulations with the uncertain initial results (cased A, G and F) are listed in Table 4, Table 5 and Table 6. Simulations with the best initial results (cases C, E and H) are listed in Table 7 and Table 8.

Table 2: Identification process based on estimate B

	x_0	x	J	σ
B1	6.4912e-004 7.3172e-004 6.4775e-003	3.9213e+003 2.9453e-007 1.0479e+004	2.0470e+010	1.2206e+003
B2	9.6489e-004 1.5761e-004 9.7059e-003	9.6767e+004 9.5282e-008 2.5423e+005	2.0469e+010	1.2206e+003
B3	1.4189e-004 4.2176e-004 9.1574e-003	8.8768e+004 1.1829e-008 1.4917e+006	2.0518e+010	1.2220e+003
B4	3.9223e-004 6.5548e-004 1.7119e-003	8.0726e+002 2.8087e-006 9.3641e+002	2.0397e+010	1.2184e+003
B5	4.8976e-004 4.4559e-004 6.4631e-003	4.1916e+003 6.4424e-007 1.4516e+004	2.0483e+010	1.2210e+003

Table 3: Identification process based on estimate D

	x_0	x	J	σ
D1	7.8023e-003 8.1126e-003 9.2939e+001	2.9786e-002 1.4805e-014 5.7847e-011	2.0514e+010	1.2219e+003
D2	1.9476e-003 2.2592e-003 1.7071e+001	7.4959e-003 2.3688e-014 6.9071e-010	2.0523e+010	1.2222e+003
D3	9.2338e-003 4.3021e-003 1.8482e+001	3.5463e-002 4.4451e-014 1.2145e-010	2.0512e+010	1.2219e+003
D4	1.1112e-003 2.5806e-003 4.0872e+001	4.2380e-003 3.0869e-014 1.3399e-009	2.0525e+010	1.2223e+003
D5	7.1122e-003 2.2175e-003 1.1742e+001	2.7386e-002 2.9793e-014 4.4736e-011	2.0515e+010	1.2220e+003

Table 4: Identification process based on estimate A

	x_0	x	J	σ
A1	2.3478e+003 3.5316e+000 8.2119e-004	9.7827e+001 4.1029e-012 9.5298e-003	6.1831e+008	2.1214e+002
A2	3.4039e+003 5.8527e+000 2.2381e-004	9.6832e+001 2.0671e-005 4.3173e-003	7.8288e+008	2.3871e+002
A3	6.9908e+003 8.9090e+000 9.5929e-004	9.7533e+001 1.3032e-002 4.5518e-002	1.2871e+008	9.6791e+001
A4	2.5751e+003 8.4072e+000 2.5428e-004	1.0659e+002 5.9861e-003 6.8127e-002	3.1766e+006	1.5206e+001
A5	3.4998e+003 1.9660e+000 2.5108e-004	9.6709e+001 7.4232e-012 1.6461e-003	8.7931e+008	2.5298e+002

Table 5: Identification process based on estimate F

	x_0	x	J	σ
F1	6.4432e+000 3.7861e+002 8.1158e+002	1.5864e+001 1.0731e+001 2.1056e-003	1.3001e+010	9.7276e+002
F2	9.6489e+000 1.5761e+002 9.7059e+002	2.3653e+001 6.7821e+000 3.2794e-003	1.0089e+010	8.5693e+002
F3	1.4189e+000 4.2176e+002 9.1574e+002	3.4946e+000 1.1493e+001 1.9938e-003	1.8693e+010	1.1664e+003
F4	3.9223e+000 6.5548e+002 1.7119e+002	9.6760e+000 1.4281e+001 1.5973e-003	1.5685e+010	1.0685e+003
F5	4.8976e+000 4.4559e+002 6.4631e+002	6.7229e+001 9.8366e+003 2.0063e-006	2.4823e+009	4.2506e+002

Table 6: Identification process based on estimate G

	x_0	x	J	σ
G1	8.7594e+000 5.5016e-002 6.2248e+003	3.3438e+001 2.0093e-010 9.8757e-003	9.2765e+009	8.2170e+002
G2	6.9908e+000 8.9090e-002 9.5929e+003	2.6686e+001 9.9225e-011 7.5063e-003	1.1175e+010	9.0189e+002
G3	2.5751e+000 8.4072e-002 2.5428e+003	9.8300e+000 2.2981e-011 1.4423e-004	1.6736e+010	1.1037e+003
G4	3.4998e+000 1.9660e-002 2.5108e+003	1.3360e+001 1.9389e-011 2.7982e-003	1.5474e+010	1.0613e+003
G5	8.3083e+000 5.8526e-002 5.4972e+003	3.1716e+001 4.9729e-012 9.2473e-003	9.7432e+009	8.4212e+002

Table 7: Identification process based on estimate C

	x_0	x	J	σ
C1	7.4469e-001 1.8896e+000 6.8678e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
C2	7.5373e-001 3.8045e+000 5.6782e-002	6.8561e+001 4.6706e+000 4.0747e-003	2.4300e+009	4.2056e+002
C3	7.7917e-001 9.3401e+000 1.2991e-002	6.7643e+001 2.9535e+001 6.6606e-004	2.4735e+009	4.2430e+002
C4	3.1122e-001 5.2853e+000 1.6565e-002	6.7297e+001 1.6407e+002 1.2009e-004	2.4807e+009	4.2492e+002
C5	6.8921e-001 7.4815e+000 4.5054e-002	6.7233e+001 1.3432e+003 1.4693e-005	2.4821e+009	4.2504e+002

Table 8: Identification process based on estimate E

	x_0	x	J	σ
E1	4.4678e+002 3.0635e-001 5.0851e-001	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
E2	1.8185e+002 2.6380e-001 1.4554e-001	1.0654e+002 6.0101e-003 6.8055e-002	3.1730e+006	1.5197e+001
E3	5.4986e+002 1.4495e-001 8.5303e-001	1.0654e+002 6.1258e-003 6.8570e-002	3.2107e+006	1.5287e+001
E4	3.3772e+002 9.0005e-001 3.6925e-001	1.9627e+002 1.0516e-009 5.3872e-001	1.7603e+009	3.5795e+002
E5	5.4701e+002 2.9632e-001 7.4469e-001	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001

Table 9: Identification process based on estimate H

	x_0	x	J	σ
H1	1.4189e+002 4.2176e-003 9.1574e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
H2	7.3033e+002 4.8861e-003 5.7853e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
H3	5.4681e+002 5.2114e-003 2.3159e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
H4	6.9483e+002 3.1710e-003 9.5022e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001
H5	3.3536e+002 6.7973e-003 1.3655e-002	1.0654e+002 6.0102e-003 6.8055e-002	3.1730e+006	1.5197e+001

The identification processes based on the worst initial results (cases B and D) lead to the different final vectors of searched parameters but the criteria function doesn't change significantly.

The identification processes based on the uncertain initial results (cased A, G and F) lead to the different final vectors of searched parameters as well as to the different value of criteria function. Moreover, one of the processes (case A4) converged to the right solution.

The identification processes based on the best initial results (cases C, E and H) consisted of two different groups. Simulation based on cases H led to the same final vector of searched parameters as well as to the same value of criteria function. On the contrary, simulations based on cases C and E had to be moved to the group of uncertain initial results.

Concerning simulations based on case C, only one process (case C1) converged to the right solution. Concerning simulations based on case E, only one process (case E4) converged to the wrong solution.

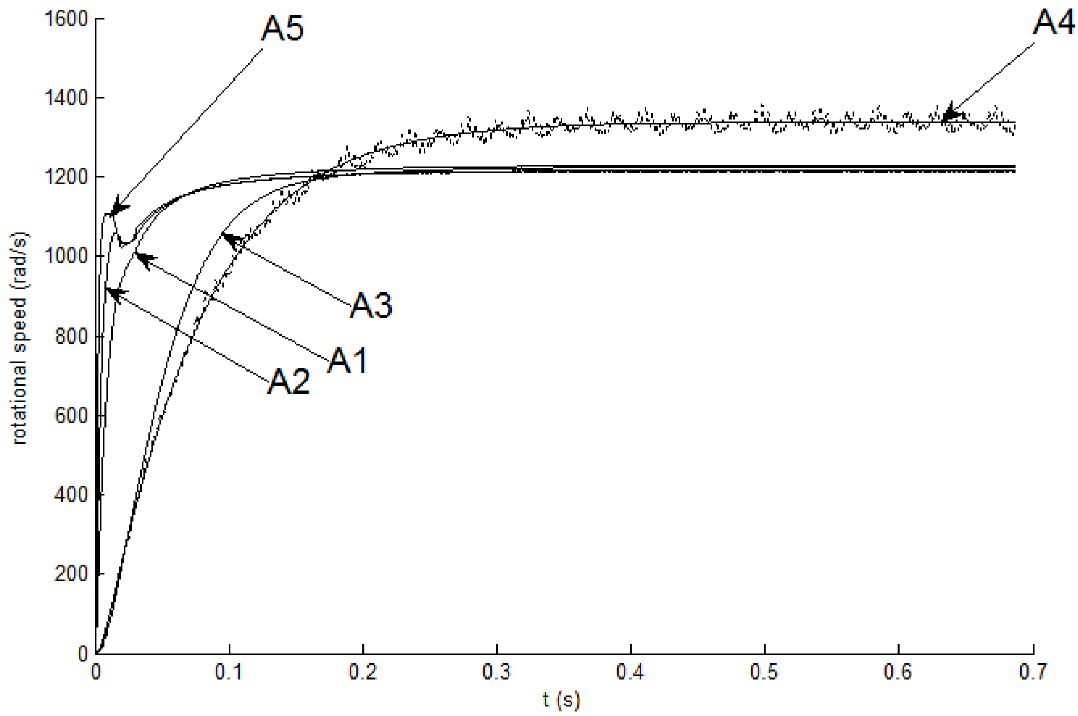


Fig. 6: Identification process: case A (using final vectors x in Table 4)

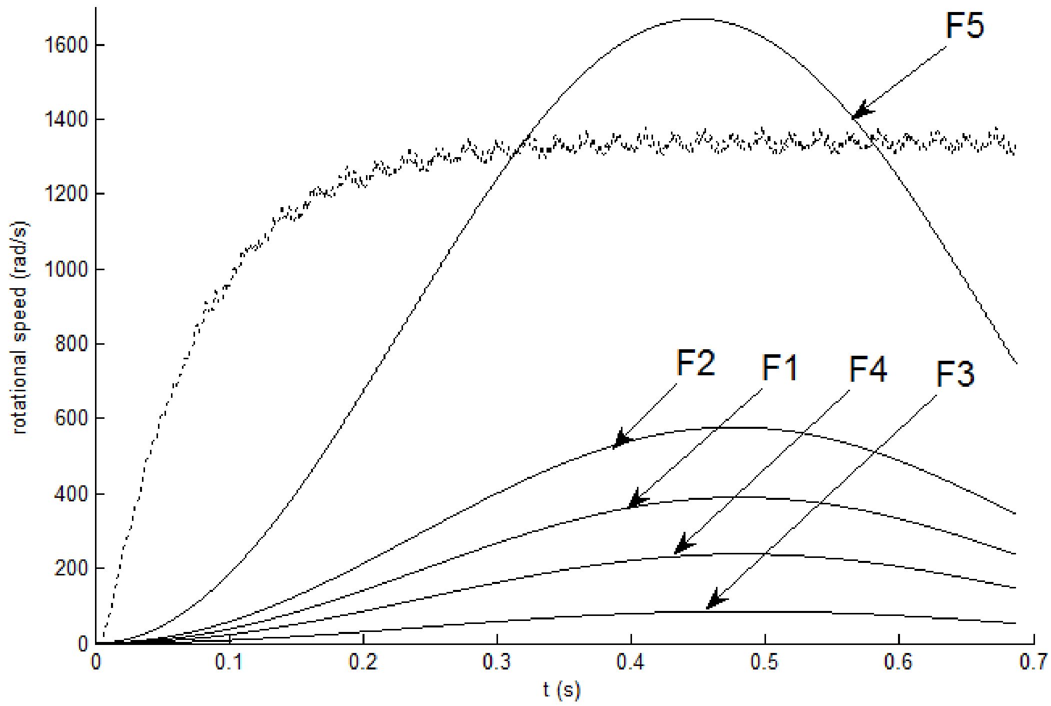


Fig. 7: Identification process: case F (using final vectors x in Table 5)

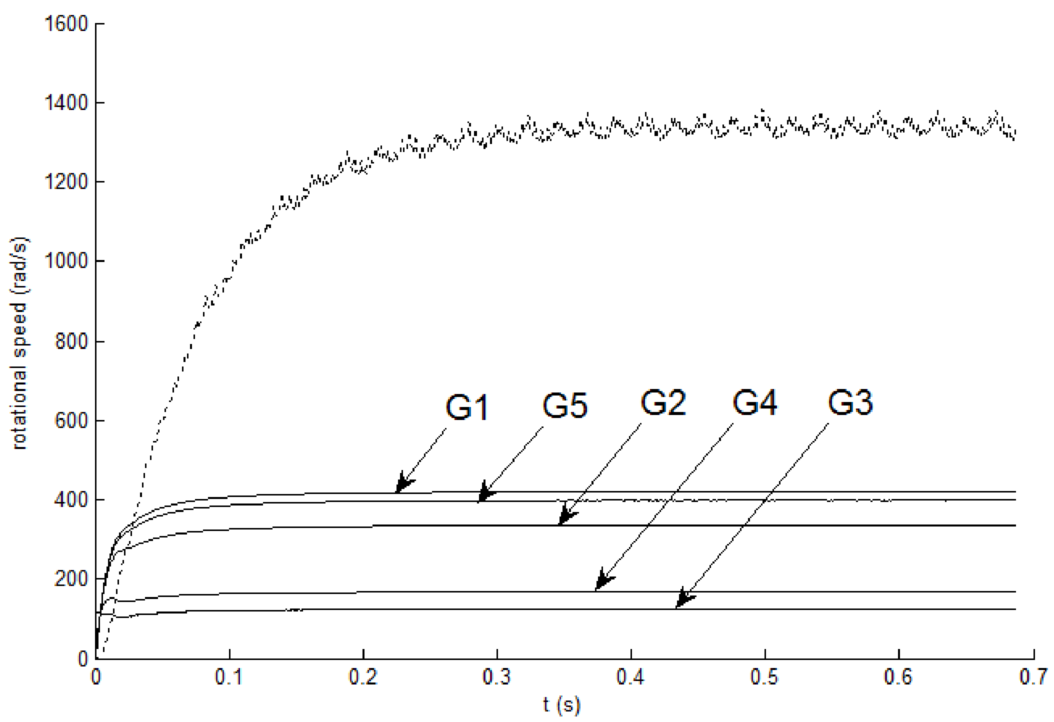


Fig. 8: Identification process: case G (using final vectors x in Table 6)

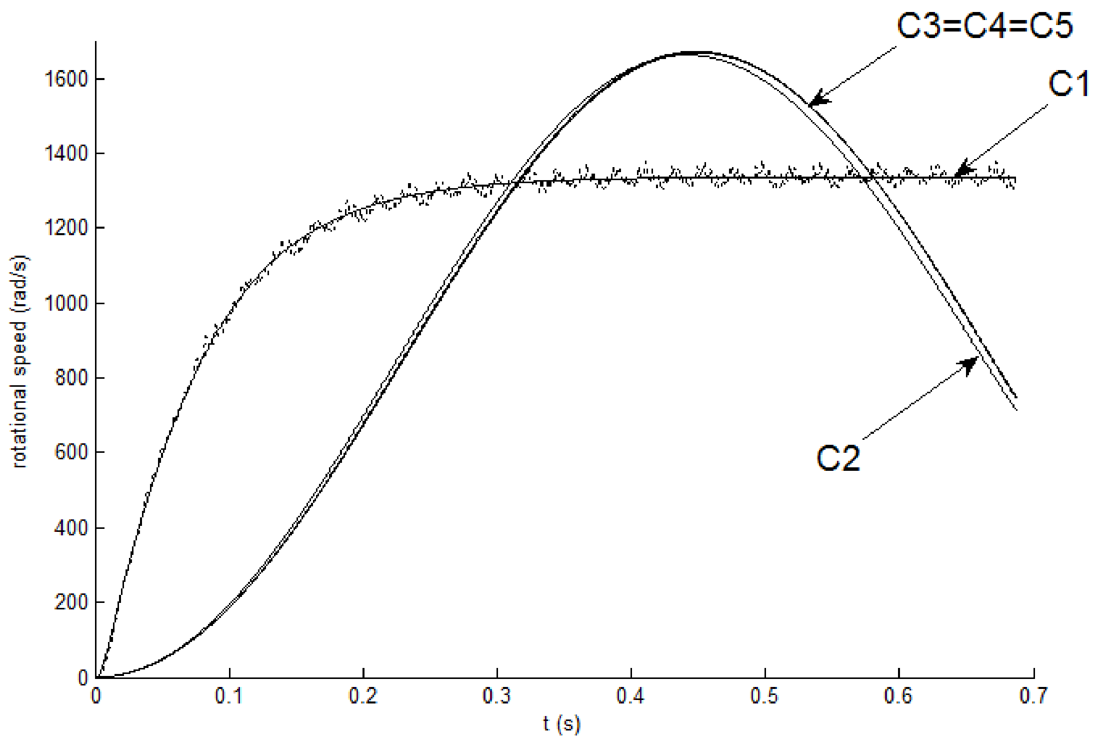


Fig. 9: Identification process: case C (using final vectors x in Table 7)

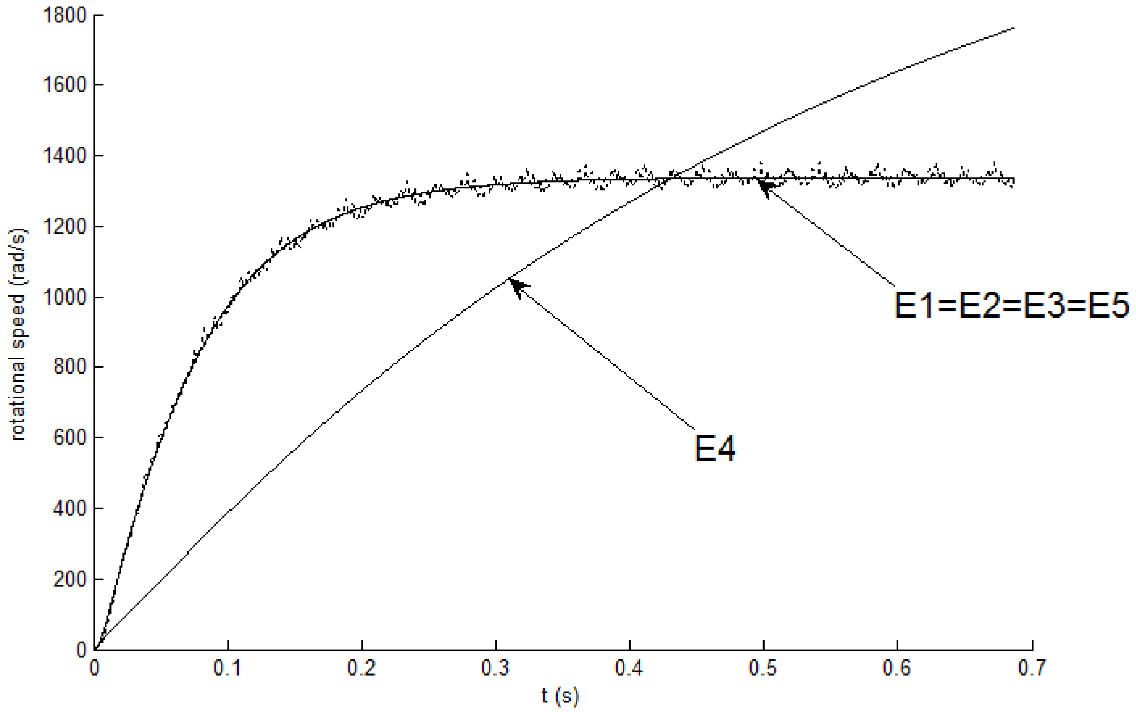


Fig. 10: Identification process: case E (using final vectors x in Table 8)

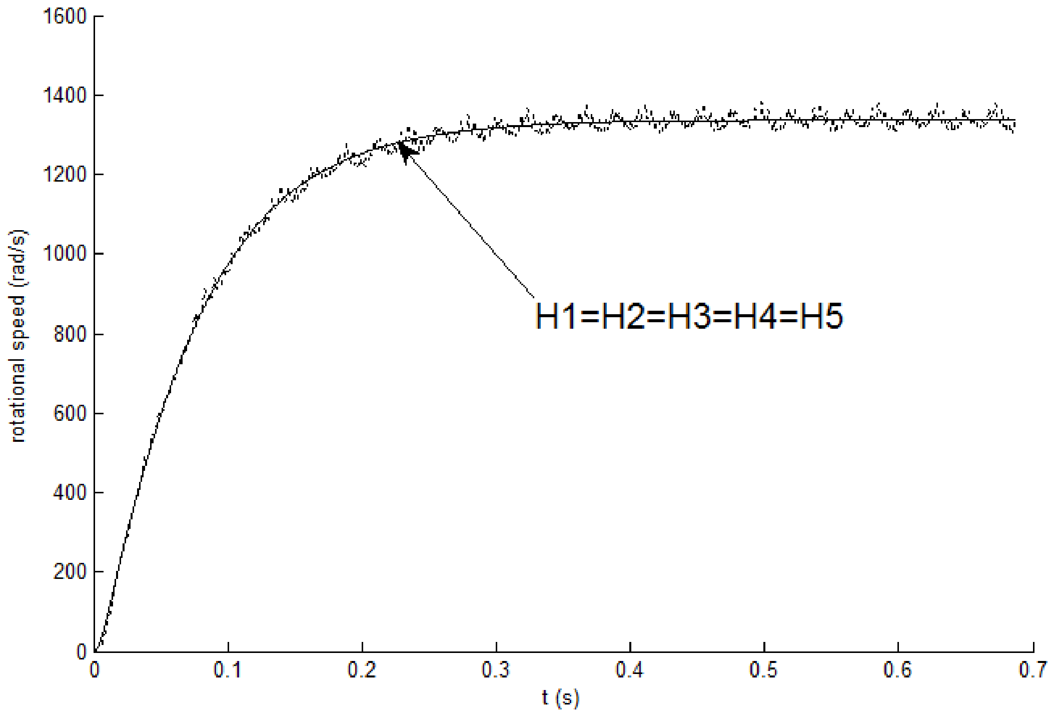


Fig. 11: Identification process: case H (using final vectors x in Table 9)

Conclusion

Experimental application of MATLAB *fminsearch* function based on Nelder-Mead Simplex Method to simple mathematical model showed excellent results. The model simulated response practically matched the real system output. It is clearly visible we can get searched parameters of the mathematical model from parameter offline identification of rotational speed step response.

The idea of operator minimal experience was confirmed. If operator sets bad initial estimation, the identification process would find another local minimum or would diverge. It is clearly visible that random initial estimate is not the most suitable method; the best way to find global minimum is to meet order of magnitude of searched parameters.

The software support of the *fminsearch* function of the MATLAB program is a powerful tool with some limitation. It is a comfortable advantage of the MATLAB program to visualize simultaneous responses of both the mathematical model simulation and the real time system that can help operator to evaluate the identification process.

Acknowledgment

The work presented in this paper has been supported by the Ministry of Defence of the Czech Republic (K206 Department development program "Complex aviation electronic system for unmanned aerial systems").

The paper has been published under the umbrella of University of Defence MATLAB Group (UDeMAG Society).

References:

[1] DUB, Michal – JALOVECKÝ, Rudolf. DC Motor Experimental Parameter Identification using the Nelder-Mead Simplex Method. In *Proceedings of EPE-PEMC 2010 - 14th International Power Electronics and Motion Control Conference.* September 6-9, 2010, Ohrid, Republic of Macedonia. Skopje : Ss Cyril and Methodius University, 2010, pp. S4-9-S4-11. ISBN 978-1-4244-7854-5. IEEE Catalog Number CFP1034A-DVD.

[2] KALOUS, Jaroslav – KRATOCHVÍL, Ctirad – HERIBAN, Pavel. *Dynamics of Rotational Electromechanical Drives (In Czech).* Brno : ÚT AVČR – Centre of Mechatronics, 2007. 119 p. ISBN: 80-214-3340-X.

[3] MODRLÁK, Osvald. *Introduction to the identification (in Czech)*. Liberec : Technical University, 2004. 31 p. Electronic textbook [online]. 2004-08-24 [cit. 2011-02-03]. Available at URL: <http://www.fm.vslib.cz/~krtsub/fm/tr1/tar1_zid.pdf>

[4] NOSKIEVIČ, Petr. *System Modelling and Identification (in Czech).* Ostrava : Montanex, 1999. 276 p. ISBN: 80-7225-030-2.

[5] *fminsearch (MATLAB Functions)* [online]. © 1984-2011 [cit. 2011-02-03]. Available at URL: < http://www.mathworks.com/help/techdoc/ref/fminsearch.html>

[6] DUB, Michal. Offline Parameter Identification - the Random Initial Estimation Issue. In *Proceedings of the 14th International Conference „Mechatronika 2011".* June 1-3, 2011, Trenčianske Teplice, Slovak Republic. Trenčín : Alexander Dubček University of Trenčín, 2011, pp. 72-75. ISBN 978-80-8075-476-1.